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BAR-DRIVEN DARK HALO EVOLUTION: A RESOLUTION OF THE CUSP–CORE CONTROVERSY

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ABSTRACT

Simulations predict that the dark matter halos of galaxies should have central cusps, while those inferred from observed galaxies do not have cusps. We demonstrate, using both linear perturbation theory and n-body simulations, that a disk bar, which should be ubiquitous in forming galaxies, can produce cores in cuspy CDM dark matter profiles within five bar orbital times. Simulations of forming galaxies suggest that one of Milky Way size could have a 10 kpc primordial bar; this bar will remove the cusp out to $\sim$5 kpc in $\sim$1.5 gigayears, while the disk only loses $\sim$8% of its original angular momentum. An inner Lindblad-like resonance couples the rotating bar to orbits at all radii through the cusp, transferring the bar pattern angular momentum to the dark matter cusp, rapidly flattening it. This resonance disappears for profiles with cores and is responsible for a qualitative difference in bar driven halo evolution with and without a cusp. This bar induced evolution will have a profound effect on the structure and evolution of almost all galaxies. Hence, both to understand galaxy formation and evolution and to make predictions from theory it is necessary to resolve these dynamical processes. Unfortunately, correctly resolving these important dynamical processes in *ab initio* calculations of galaxy formation is a daunting task, requiring at least 4,000,000 halo particles using our SCF code, and probably requiring many times more particles when using noisier tree, direct summation, or grid based techniques, the usual methods employed in such calculations.

Subject headings: galaxies:evolution — galaxies: halos — galaxies: kinematics and dynamics — cosmology: theory — dark matter

1. INTRODUCTION

The cold dark matter (CDM) model for structure formation has had a wide range of successes in explaining the observed universe. However, particularly at small scales, several vexing problems remain. Perhaps the most widely discussed shortcoming concerns the central density profiles of galaxies. Cold dark matter structure formation simulations predict a universal cuspy halo profile (Navarro et al. 1997, hereafter NFW). This profile, $\rho \propto r^{-\gamma}(1+r/r_s)^{\gamma-3}$ or $\rho \propto r^{-\gamma}(1+(r/r_s)^{3-\gamma})^{-1}$ with $\gamma = 1$, was first presented by NFW based on a suite of collisionless n-body simulations with different initial density fluctuation spectra and cosmological parameters. More recent work debates the value of γ (Moore et al. 1998; Jing and Suto 2000), but most estimates are in the range $1 < \gamma < 1.5$. Even before the discovery of a universal density profile, several authors pointed out the apparent discrepancy between the cuspy central density profiles predicted by dissipationless numerical simulations and those inferred from the rotation curves of galaxies (Moore 1994; Flores and Primack 1994; Burkert 1995). Although some recent observational evidence claims a marginal consistency with such cuspy dark matter profiles (van den Bosch and Swaters 2001), most do not (Côté et al. 2000; de Blok et al. 2001; Blais-Ouellette et al. 2001).

This led to a flurry of papers trying to explain the discrepancy. Given the numerous other successes of the CDM model most of these explanations involved altering the model only at small scales, which would only affect the cuspiness of the dark matter density profile. Some changed the fundamental nature of the dark matter particle itself: collisional dark matter (Spergel and Steinhardt 2000), decaying dark matter (Cen 2001), fluid dark matter (Peebles 2000), repulsive dark matter (Goodman 2000), and annihilating dark matter (Kaplinghat et al. 2000). Others changed the temperature of the dark particle from cold to warm to reduce the small scale power, i.e. warm dark matter (Hogan and Dalcanton 2000; Colín et al. 2000; Eke et al. 2001; Ávila-Reese et al. 2001; Bode et al. 2001), while others altered the shape of the spectrum at small scales by changing inflation (Kamionkowski and Liddle 2000). At the most extreme, some authors suggested that the only solution was to change the nature of gravity itself (de Blok and McGaugh 1998).

The simulations that predicted the existence of the central dark matter cusp (Navarro et al. 1997; Moore et al. 1998; Jing and Suto 2000) only included the dark matter component. By using rotation curve analyses and light profiles, the same techniques used to infer the above mentioned discrepancy, we know that the central regions of most galaxies are dominated by a baryonic component (van Albada and Sancisi 1986). The dynamics of the baryonic component can be quite different than that of dark matter owing to its dissipative nature: baryons can shock and cool. Furthermore, even a dissipationless, stellar baryonic component can have a markedly different geometry than pure dark matter since it originally formed from a dissipative component. These different dynamical properties and the subsequent interactions of the baryonic component with the dark matter could greatly affect the dark matter structure.

Since it is commonly assumed that the addition of a

baryonic component would cause the dark matter to adiabatically contract (Blumenthal et al. 1986) thereby exacerbating the observed discrepancy, the complications caused by the baryonic component are usually dismissed. Even so, both Binney et al. (2001) and El-Zant et al. (2001) discuss mechanisms by which the baryonic component might produce a core. Much of the baryonic component will cool and likely form a disk (Fall and Efstathiou 1980). Disk bars and strong grand-design structure can strongly interact with the dark matter (Weinberg 1985; Hernquist and Weinberg 1992) by transferring angular momentum to the spheroid-halo component. Quoting from Hernquist and Weinberg (1992)
"Cosmological simulations of [dark] halo formation from plausible initial conditions generally yield halos whose density structure resembles that of [a Hernquist profile] (Dubinski and Carlberg 1991). While somewhat uncertain, it would appear that this result is in conflict with observed rotation curves of at least some galaxies where the dark matter halos apparently have significant core regions with roughly constant density (e.g. Flores et al. (1993)). The simulations reported here [of rotating bars] provide, in principle, a mechanism for developing cores in halos long after they form."
It is this mechanism that we explore in this paper. We present a brief description and simulations of the mechanism in §2 and in §3 we discuss the dynamics in more detail along with other implications.

2. THE MECHANISM

In an equilibrium dark matter halo the average orbit is highly eccentric ($e \approx 0.5$ on average) and, therefore, has low angular momentum for its energy. Very little energy is required to remove the cusp; a minor addition of angular momentum provides enough circularization to exclude an eccentric orbit from the central region. The pattern of a rotating bar has enough angular momentum, if transferred to the cusp, to remove it altogether. Our basic picture is as follows:

1. Start with a forming galaxy. CDM simulations predict that its dark matter distribution will be cuspy (e.g. an NFW profile).

2. As the galaxy continues to form, smooth accretion of gas and stochastic merging and dissipation is followed by the settling of the baryons into a mostly gaseous, cold disk.

3. As the disk becomes more massive and centrally concentrated, at some point it will overwhelm the halo support and form a bar as seen in simulations of galaxy formation that include a dissipative baryonic component (Katz and Gunn 1991; Steinmetz 1997). Since the disk is mostly gaseous and cold and the halo is quite disturbed, it makes it easy to form a very strong bar. Simulations suggest that for Milky Way sized galaxies early bars have semi-major axes approaching 10 kpc (Steinmetz 1997).

4. The rotating bar pattern excites a gravitational wake in the dark matter. The wake trails the bar causing the bar to slow its rotation and transfer its angular momentum to the dark matter (Weinberg 1985). This forms a core in the dark matter distribution. The 10 kpc primordial bar will remove the cusp out to ∼5 kpc in ∼1.5 gigayears.

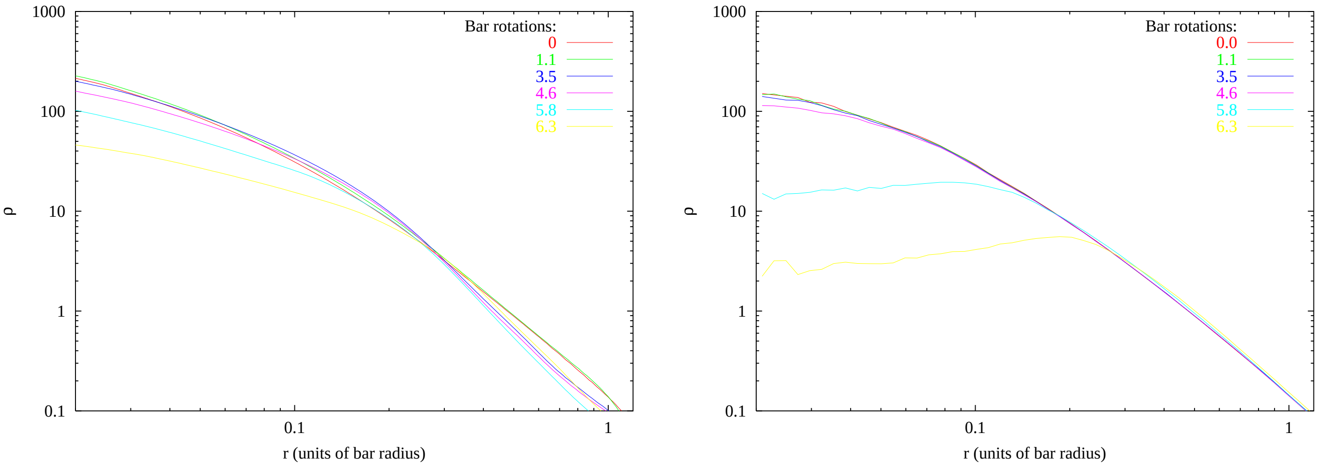
5. After the formation of the core early on, a *classic* stellar bar may form and experience low torque in the current epoch.

It is easy to estimate that a strong bar can have important consequences for halo evolution. A non-axisymmetric bar force is dominated by its quadrupole component. A toy model for a rotating gravitational quadrupole is two masses in orbit at the same distance from the center of a galaxy but at opposite position angles. We can use the Chandrasekhar dynamical friction formula to estimate the time scale for transferring all of the bar's angular momentum to the halo. The answer is a few bar rotation periods (see Weinberg 1985). Since the total angular momentum in the bar approaches that of the dark matter halo within the optical radius, we conclude that the evolution of the dark matter halo induced by a bar can be significant. These estimates have been further refined by Hernquist and Weinberg (1992) and Debattista and Sellwood (1998, 2000) using n-body simulations.

The evolution of the halo may be estimated analytically using a perturbation expansion of the collisionless Boltzmann equation and a solution of the resulting initial value problem. The zeroth-order solution specifies the equilibrium galaxy and the first-order solution determines the linear response of the galaxy to some perturbation. The second-order solution contains the first non-transient change in the underlying distribution. By taking the limit for the evolution time scale to be much larger than an orbital time, the transient contribution can be made arbitrarily small. However, we are often interested in intermediate time scales. Explicit comparisons with n-body simulations shows that this approximation is acceptable even for a small number of orbital time scales. Mathematical details can be found in Weinberg (1985, 2001b).

Figure 1a shows the perturbation theory prediction for the evolution of the density profile. The simulation is of a bar in an NFW profile with a concentration of 20. The disk contains half the mass within the bar radius and the bar contains 30% of the disk mass. We choose the corotation radius to be the NFW scale length and the bar radius is chosen to be 0.5 scale lengths. The results described below are only weakly sensitive to this choice. The bar figure is represented by a homogeneous ellipsoid with axis ratios of 1:0.5:0.05. We derive the bar force from the quadrupole term of the gravitational potential of the ellipsoid. Ignoring the higher order multipoles in the bar potential does not greatly change the results but does cause us to slightly underestimate the evolution (Hernquist and Weinberg 1992).

The angular momentum transfer to the dark matter takes place at commensurabilities between the pattern speed and orbital frequencies and is dominated by a few strong resonances. The torque pushes the inner halo orbits to higher angular momentum and energy, removing gravitational support for the inner cusp. The halo expands and flattens the cusp. After a few bar rotation times (several



(a) Linear theory

(b) Simulation

FIG. 1.— Evolution of an NFW profile with an embedded disk bar. Profiles are a time sequence labeled by the number of bar rotation times as predicted from linear perturbation theory (a) and from a n-body simulation (b).

hundred million years) the central cusp is flattened, but not enough to match the observations.

Although the perturbative approach yields more accurate results, free from the numerical noise inherent in n-body simulations, they are only valid in the linear regime. Hence, we perform n-body simulations using a parallel implementation with the Message Passing Interface (MPI), of the algorithm described in Weinberg (1999), a self consistent field (SCF) technique. This algorithm defines a set of orthogonal functions whose lowest-order member is the unperturbed profile itself. Each additional member in the series probes successively finer scales. Because all scales of interest here can be represented with a small number of degrees of freedom, the particle noise is low.

The initial conditions are a Monte Carlo realization of the exact isotropic phase space distribution function for the NFW profile, determined by Eddington inversion (see Binney and Tremaine 1987, Chap. 4), using 4,000,000 equal mass particles. As we discuss below this number of particles is sufficient to give a converged result. Furthermore, when simulated without a bar disturbance this realization does not evolve and remains in equilibrium. The rotating bar disturbance, with the same strength and size as in the perturbative calculation, is again represented by the quadrupole term of gravitational potential of the ellipsoid. It is turned on adiabatically over four bar rotation times to avoid sudden transients. The use of the quadrupole term only, the first contributing multipole after the monopole, ensures that the dark matter halo remains in approximate equilibrium as the bar perturbation is applied. The early n-body evolution, shown in Figure 1b, is similar to the results from the perturbative approach. The inner evolution follows the analytic predictions up to five bar rotation times. At this point, approximately 30% of the available angular momentum in the bar pattern has been transferred to the halo. Subsequent evolution is more rapid, presumably due to the non-linear response of the near-resonant orbits, although the details remain to be investigated. A similar super-linear increase in torque was reported by Hernquist and Weinberg (1992). The cusp within ~ 0.5 bar radii is completely removed after only 7 bar rotations. At this time the bar pattern has lost all of its original angular momentum, the stars in the bar have lost $\sim 25\%$ of their angular momentum, and the disk as a whole has lost only $\sim$8% of its original angular momentum, and the disk as a whole has lost only $\sim$8% of its original angular momentum.

3. DISCUSSION

3.1. *Dynamics*

In the previous section we mentioned that one can estimate the dynamical friction decay time of a rotating bar by applying the Chandrasekhar dynamical friction formula to a toy model for the bar consisting of two masses in orbit at the same distance from the center of the galaxy but at opposite position angles. This physical argument is a gross simplification, however. The Chandrasekhar dynamical friction formula is derived by considering the momentum transfer of stars gravitationally scattered by a traveling body. The problem is much more complicated in a halo where individual orbits are quasiperiodic. The gravitational wake, i.e. the response of the dark halo to the bar, is not as simple as the Chandrasekhar formula would lead one to believe. A given orbit may encounter the rotating perturbation many times. If the precession frequency leads or trails the pattern, the net torque applied to this orbit will be zero on average. This appears to be in conflict with dynamical friction but really it is not. The bar will only receive a net torque if the gravitational wake either trails or leads the bar pattern and this can only occur if some of the orbital actions are changed, something that can only occur at or near resonances. At exact commensurabilities between the orbital and pattern frequencies individual orbits receive kicks breaking symmetry, resulting in a density response that trails the bar. To have this symmetry broken requires gradients in the phase space density at the positions of these resonances.

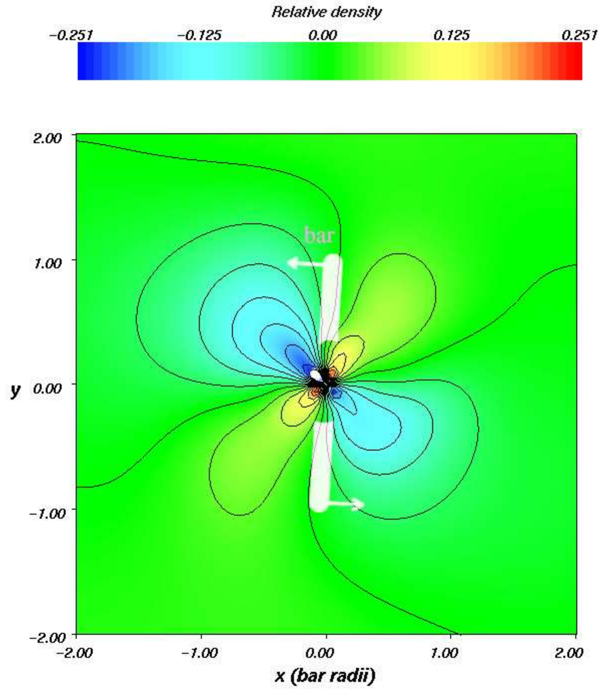


FIG. 2.— The halo response and bar position during the evolution of an NFW model halo. We subtract the mean density and plot the amplitude of the resulting wake where white represents underdense regions and black overdense regions.

In Figure 2, we plot the actual gravitational wake induced by a bar in an n-body simulation. It is similar to the simulations described above but uses an NFW halo realized with 10^7 equal mass particles and is plotted after 5 bar pattern rotations. This figure shows the halo density distortion in response to the bar. The bar's size and phase are shown schematically. The bar pattern leads the wake and therefore the bar torques the halo.

The location in the halo of the peak angular momentum transfer depends on the halo profile itself in two ways. First, the torqued orbits will be near commensurabilities between the orbital frequencies and the pattern speed. These commensurabilities or *resonances* take the form $l_r\Omega_r + l_\phi\Omega_\phi = m\Omega_{bar}$ where the three values of Ω are the radial, azimuthal and pattern frequencies, respectively, l_r and l_ϕ are integers and m is the azimuthal multipole index. For the $l = m = 2$ multipole, we will denote a particular resonance by the tuple (l_r, l_ϕ). Low-order (high-order) resonances have small (large) values of $|l_r|$ or $|l_\phi|$. Second, a net torque requires a differential in phase space density on either side of a particular resonance. It is not the dark matter density of the inner halo but a large phase space gradient that is key to a large bar torque. In fact, if the bar were located in an infinitely large homogeneous core, no matter how dense, there would be no net torque, in contrast to the predictions of the simple toy model. If the bar is inside the halo core, then there will be very few nearby resonances with a differential in phase space density and the torque would be diminished; the dominant resonant orbits in this case would be at or beyond the core radius. Conversely, if the bar is in a dark matter cusp, orbits near and inside the bar radius cover a large range of frequencies. There are low-order resonances deep in the cusp where the phase space gradient is large. The torqued cusp orbits move to larger radii, decreasing the cusp density and decreasing the overall depth of the potential well. Both of these effects cause the cusp to expand overall. Thus, the formation of a bar will naturally eliminate an inner cusp.

Linear theory allows us to explicitly identify the dominant resonances and the angular momentum transfer to halo orbits for models both with and without cores. The overall torque is dominated by one or two resonances in each case. For the NFW profile, the torque is dominated by the resonance $(-1, 2)$; this is analogous to an inner Lindblad resonance. This resonance does not occur for a comparable King (1966) model. The first contributing resonance for the King model is $(2, -2)$ near the location of the core radius. Higher-order resonances that occur at larger radii are well confined to a single characteristic radius (close to vertical in Fig. 3. However, the $(-1, 2)$ resonance is unique. It always exists in a cusp as $r \to 0$ because $\Omega_r \to 2\Omega_\phi$ as the orbital angular momentum J approaches zero. Therefore there is always some value of J for which $-\Omega_r+2\Omega_\phi = 2\Omega_{bar}$ even though Ω_r and Ω_ϕ both diverge for small r. For this reason, the $(-1, 2)$ resonance can affect orbits deep within a cusp, dramatically changing the inner profile. For a model with a core, the core expands somewhat as angular momentum is transferred toward the outer halo but otherwise remains qualitatively similar to its initial state.

Since bar instability is ubiquitous in self-gravitating

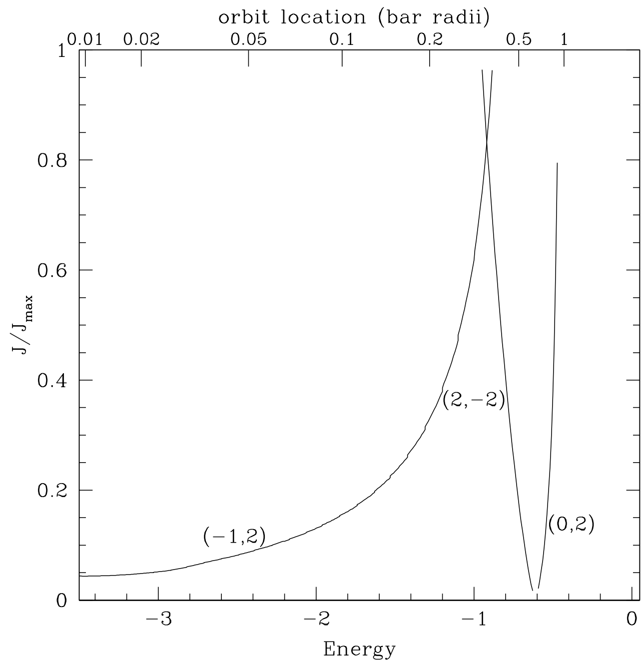


FIG. 3.— Location of low-order resonances in energy (lower axis) and characteristic radius (upper axis) for the bar in the NFW profile. The vertical axis describes the orbital angular momentum J in units of the maximum angular momentum for a given energy, J_{max}. The inner Lindblad-like resonance extends throughout the inner cusp. This resonance is absent from dark halo models with cores.

disks, continued accretion and cooling is likely to precipitate a bar instability in the forming inner gas disk early on. Once the rotating bar forms, its body angular momentum can be transferred to the halo as described above, flattening and removing the inner cusp. It is possible for this process to occur in stages with multiple bars; at each stage the inner core will grow. There will be a transition in the magnitude of the torque and a slowing of halo evolution when the inner $(-1, 2)$ resonance is finally eliminated.

3.2. *Implications*

Because the bar–cusp coupling depends on near-resonant dynamics, simulations with very high resolution will be required to resolve the dynamics properly. We are confident in our n-body results since they agree with the exact perturbative results in the linear regime, once we have enough particles to reach convergence. To test for convergence we ran a suite of simulations with increasing particle number from 10^4 through 4×10^7 and determined that, for our SCF expansion method, we obtained convergence in the evolution for particle numbers $\gtrsim 4\times10^6$. This class of potential solver (see Clutton-Brock 1972, 1973; Kalnajs 1976; Fridman and Polyachenko 1984; Hernquist and Ostriker 1992) suppresses small scale fluctuations. Direct summation approaches (e.g. Kawai et al. 2000, GRAPE), tree codes (Barnes and Hut 1986) and grid based codes (Sellwood and Merritt 1994; Pearce and Couchman 1997) have more inherent small scale noise and will most likely require even higher particle numbers to obtain the same convergence. Note that even if the resonances are not resolved, these simulations will still exhibit significant torque. Our suite of simulations shows that the overall torque *increases* as the particle number *decreases*[1]! These same simulations give us good agreement with Chandrasekhar's formula. Such agreement is not a good indication that one is observing the correct dynamics. Conversely, the Chandrasekhar formula works well in simulations with low to moderate particle number because the resonances are obliterated by artificial diffusion and, therefore, are well represented by simple scattering. In short, it is difficult to see resonant effects in n-body simulations because the diffusion rate is high for moderate numbers of particles. Astronomical sources of noise such as orbiting substructure, decaying spiral waves, lopsidedness, etc. do not produce enough small scale noise to affect this resonant evolution but instead produce large scale deviations from equilibrium that will not drive significant relaxation in the inner halo within several orbital times (see Weinberg 2001b,a, for estimates of these timescales).

The evolution of bars within dark halos has recently been studied by Debattista and Sellwood (1998, 2000). They simulated bars in dark halos by constructing a strong bar from a Q-unstable disk and then following its evolution. They found that the bars rapidly transferred angular momentum to the massive (non-rotating) halos as predicted by Weinberg (1985). On the other hand, a few direct and a wider variety of indirect inferences from observations indicate that galaxies today harbor rapidly rotating bars. Hence, Debattista & Sellwood concluded that dark matter halos must have low density (or large cores) to be consistent with observations and could not have cusps. These results are consistent with our scenario. As the disk matures and becomes stellar rather than gas dominated, a normal stellar bar may form through secular growth or instability. The first generation of bar evolution would have eliminated the inner halo torque by removing the cusp and, hence, be consistent with the Debattista and

[1] If the orbit diffusion becomes large enough, it is plausible that the torque will decrease to zero, however.

Sellwood (2000) arguments.

The bar torque will affect the kinematics of the bulge. Kormendy (1982) finds that bulges of SB galaxies have sufficient spin to be rotationally flattened. However, triaxial galaxy bulges appear to be kinematically midway between an isothermal and rotationally-supported disk component. The bulge length scale is smaller the *classic* bar length and much smaller than the length of a strong primordial bar. Because the bulge profile is not supported by relatively low-energy eccentric orbits, the bulge will not be subject to the strong density evolution predicted here.

Most of the evidence against central dark matter cusps in galaxies concerns dwarf galaxies, particularly those with low surface brightness (Côté et al. 2000; de Blok et al. 2001; Blais-Ouellette et al. 2001) since these systems offer more precise rotation curve decompositions. One might not expect strong bars to form in such systems and, hence, for the mechanism proposed here not to have much relevance. However, the same analysis used to indicate the lack of a central dark matter cusp also shows that these low surface brightness galaxies are deficient in baryons by three or four times compared to normal galaxies (van den Bosch and Swaters 2001). If a strong bar forms in a gas rich disk of a dwarf galaxy, much of the gas will lose substantial amounts of angular momentum and be driven towards the center (Roberts et al. 1979). This dense cold gas would then be expected to undergo a strong starburst and, owing to the shallow potential well of the dwarf galaxy, much of the gas would be expelled as a wind (Dekel and Silk 1986). The remaining galaxy would be one of low surface brightness possessing a core in its dark matter distribution. Our bar mechanism, therefore, not only provides a natural explanation for the existence of dark matter cores but for the existence of low surface brightness dwarfs as well; those dwarf galaxies that had the strongest bars will have the largest cores and lowest surface brightness. There is also some evidence that larger, high surface brightness galaxies also have cores in their central dark matter distributions, although such determinations are much more difficult (Debattista and Sellwood 2000; Binney and Evans 2001).

Even if our proposed bar mechanism does not explain the lack of observed central dark matter cusps in all cases, it will still have a profound effect on the structure and evolution of almost all galaxies. Hence, both to understand galaxy formation and evolution and to make predictions from theory it is necessary to resolve these dynamical processes. For example: it changes the predicted dark matter densities at the solar radius and would change the predicted dark matter detection rates (see Stiff et al. 2001, and references therein), it could change the rotation velocities used in Tully-Fisher predictions and perhaps improve the agreement of theory with observations (Navarro and Steinmetz 2000); it could change disk scale lengths (Steinmetz and Navarro 1999); and it could make satellite galaxies easier to tidally disrupt by decreasing their central densities (Moore et al. 1999). Some of these consequences are qualitatively discussed in Binney et al. (2001) but it is hard to address many of these issues quantitatively with the simplified simulations presented here. In the future, we plan to perform simulations with self consistent disks and bars, eventually including gas dynamics, to investigate these issues.

The lack of an observed central dark matter cusp in galaxies is a consequence of simple dynamical evolution and does not require a fundamental change to the nature of dark matter or galaxy formation. The difference in evolutionary end states may be the result of strong star formation feedback evacuating the more weakly bound central potentials, lack of strong accretion events and mergers after the primordial bar has disappeared, or a combination of the two. Indeed, such stochastic processes naturally predict the inferred dispersion in present day profiles. However, to correctly resolve these important dynamical processes in *ab initio* calculations of galaxy formation remains a daunting task, requiring at least 4,000,000 halo particles using our SCF code, and requiring many times more particles when using noisier tree, direct summation, or grid based techniques—the usual methods employed in such calculations.

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REFERENCES

Avila-Reese, V., Colín, P., Valenzuela, O., D'Onghia, E., and Firmani, C. 2001, ApJ, 559, 516
Barnes, J. E. and Hut, P. 1986, Nature, 324, 446
Binney, J., Gerhard, O., and Silk, J. 2001, MNRAS, 321, 471
Binney, J. and Tremaine, S. 1987, *Galactic Dynamics*, Princeton University Press, Princeton, New Jersey
Binney, J. J. and Evans, N. W. 2001, in *MNRAS (submitted)*
Blais-Ouellette, S. ., Amram, P., and Carignan, C. 2001, AJ, 121, 1952
Blumenthal, G. R., Faber, S. M., Flores, R., and Primack, J. R. 1986, ApJ, 301, 27
Bode, P., Ostriker, J. P., and Turok, N. 2001, ApJ, 556, 93
Burkert, A. 1995, ApJ, 447, L25
Côté, S. ., Carignan, C., and Freeman, K. C. 2000, AJ, 120, 3027
Cen, R. 2001, ApJ, 546, L77
Clutton-Brock, M. 1972, Astrophys. Space. Sci., 16, 101
Clutton-Brock, M. 1973, Astrophys. Space. Sci., 23, 55
Colín, P., Avila-Reese, V., and Valenzuela, O. 2000, ApJ, 542, 622
de Blok, W. J. G. and McGaugh, S. S. 1998, ApJ, 508, 132
de Blok, W. J. G., McGaugh, S. S., Bosma, A., and Rubin, V. C. 2001, ApJ, 552, L23
Debattista, V. P. and Sellwood, J. A. 1998, ApJL, 493, L5
Debattista, V. P. and Sellwood, J. A. 2000, ApJ, 543, 704
Dekel, A. and Silk, J. 1986, ApJ, 303, 39
Dubinski, J. and Carlberg, R. G. 1991, ApJ, 378, 496
Eke, V. R., Navarro, J. F., and Steinmetz, M. 2001, ApJ, 554, 114
El-Zant, A., Shlosman, I., and Hoffman, Y. 2001, in *"The central kpc of starbursts and AGN: the La Palma connection", Eds. J.H. Knapen, J.E. Beckman, I. Shlosman and T.J. Mahoney, ASP conf. series, in press (2001)*, 8327
Fall, S. M. and Efstathiou, G. 1980, MNRAS, 193, 189
Flores, R., Primack, J. R., Blumenthal, G. R., and Faber, S. M. 1993, ApJ, 412, 443
Flores, R. A. and Primack, J. R. 1994, ApJ, 427, L1
Fridman, A. M. and Polyachenko, V. L. 1984, *Physics of Gravitating Systems*, Vol. 2, 282, Springer-Verlag, New York
Goodman, J. 2000, New Astronomy, 5, 103
Hernquist, L. and Ostriker, J. P. 1992, ApJ, 386, 375
Hernquist, L. and Weinberg, M. D. 1992, ApJ, 400, 80
Hogan, C. and Dalcanton, J. 2000, Phys. Rev. D, D62, 817
Jing, Y. P. and Suto, Y. 2000, ApJL, 529, L69
Kalnajs, A. J. 1976, ApJ, 205, 745
Kamionkowski, M. and Liddle, A. 2000, Phys. Rev. Lett., 84, 4525

Kaplinghat, M., Knox, L., and Turner, M. 2000, Phys. Rev. Lett., 85, 3335

Katz, N. and Gunn, J. E. 1991, ApJ, 377, 365

Kawai, A., Fukushige, T., Makino, J., and Taiji, M. 2000, PASJ, 52, 659

King, I. R. 1966, AJ, 71, 64

Kormendy, J. 1982, ApJ, 257, 75

Moore, B. 1994, Nature, 370, 629

Moore, B., Ghigna, S., Governato, F., Lake, G., Quinn, T., Stadel, J., and Tozzi, P. 1999, ApJ, 524, L19

Moore, B., Lake, G., and Katz, N. 1998, ApJ, 495, 139 p

Navarro, J. F., Frenk, C. S., and White, S. D. M. 1997, ApJ, 490, 493

Navarro, J. F. and Steinmetz, M. 2000, ApJ, 538, 477

Pearce, F. R. and Couchman, H. M. P. 1997, New Astronomy, 2, 411

Peebles, P. J. E. 2000, ApJ, 534, L127

Roberts, W. W., Huntley, J. M., and van Albada, G. D. 1979, ApJ, 233, 67

Sellwood, J. A. and Merritt, D. 1994, ApJ, 425, 530

Spergel, D. N. and Steinhardt, P. J. 2000, Physical Review Letters, 84, 3760

Steinmetz, M. 1997, in *The Early Universe with the VLT*, 156

Steinmetz, M. and Navarro, J. F. 1999, ApJ, 513, 555

Stiff, D., Widrow, L. M., and Frieman, J. 2001, Phys. Rev. D, 64, 083516

van Albada, T. S. and Sancisi, R. 1986, Royal Society of London Philosophical Transactions Series, 320, 447

van den Bosch, F. C. and Swaters, R. A. 2001, MNRAS, 325, 1017

Weinberg, M. D. 1985, MNRAS, 213, 451

Weinberg, M. D. 1999, AJ, 117, 629

Weinberg, M. D. 2001a, *Noise-driven evolution in stellar systems: A universal halo profile*, MNRAS, in press

Weinberg, M. D. 2001b, *Noise-driven evolution in stellar systems: Theory*, MNRAS, in press